W71-00621

16050DJK 08/70

PB195544

CIVIL ENGINEERING REPORT NO. 18.1
FACULTY OF ENGINEERING AND APPLIED SCIENCES
STATE UNIVERSITY OF NEW YORK AT BUFFALO

W71-ØØ621

DYNAMIC MODEL STUDY OF LAKE ERIE

Part I. Similitude Criteria and Experimental Set-Up

by

Ralph R. Rumer, Jr.

August 1970

Reproduced by
NATIONAL TECHNICAL
INFORMATION SERVICE
Springfield, Va. 22151

Prepared under the Support of the
Federal Water Pollution Control Administration
Department of the Interior (Grant No. WP-00837)

56

1 Accession Number	2 Subject Field & Group	SELECTED WATER RESOURCES ABSTRACTS INPUT TRANSACTION FORM
W71-00621	02H, 05G, 5B	

5 Organization

State University of New York, Buffalo,

6 Title

DYNAMIC MODEL STUDY OF LAKE ERIE, Part I. Similitude Criteria and Experimental Set-Up,

10 Author(s)

Rumer, Ralph R, Jr.

16 Project Designation FWQA Program Grant WP-00837.

21 Note

Available from the Clearinghouse as PB 195544, $3 in paper copy, 65¢ in microfiche

22 Citation Civil Engineering Report No. 18.1, August, 1970. 43p, 9 fig, 21 ref,

23 Descriptors (Starred First)

*Lake Erie, *Model studies, *Hydraulic models, *Simulation analysis, Currents (water), *Hydraulic similitude, Synthetic hydrology, Water circulation, Movement, Water quality, Water pollution control, Dilution,

25 Identifiers (Starred First)

Rotating model,

27 Abstract

Both analytical and experimental investigations are employed to provide basic information concerning water movement in the Lake. The knowledge and experience acquired in the course of this study should improve our capability to construct and operate rotating hydraulic models. This information should also assist in the overall water quality management of the Lake. The studies reported here deal with the idealized conditions of zero wind stress (including the inflow of the Detroit River and the outflow of the Niagara River) and, under this same flow condition, with the superposition of a uniform westerly wind over the surface of the Lake. The water mass is isothermal and variations in density are assumed absent. Under these conditions, investigations of the circulation patterns were conducted. Also studied was the oscillation of the entire water mass in the model lake. In particular, attention was given to the transient effect that this mass oscillation had on the otherwise steady-state circulation pattern and its effect on the mixing and dilution of waste discharges into the lake. The mixing and dilution of tracers introduced at various positions in the model lake were also studied under these same conditions of zero wind stress and constant throughflow and with the superposition of the westerly wind. (USGS Abstract)

25 Identifiers (Starred First)

Rotating model,

27 Abstract

Both analytical and experimental investigations are employed to provide basic information concerning water movement in the Lake. The knowledge and experience acquired in the course of this study should improve our capability to construct and operate rotating hydraulic models. This information should also assist in the overall water quality management of the Lake. The studies reported here deal with the idealized conditions of zero wind stress (including the inflow of the Detroit River and the outflow of the Niagara River) and, under this same flow condition, with the superposition of a uniform westerly wind over the surface of the Lake. The water mass is isothermal and variations in density are assumed absent. Under these conditions, investigations of the circulation patterns were conducted. Also studied was the oscillation of the entire water mass in the model lake. In particular, attention was given to the transient effect that this mass oscillation had on the otherwise steady-state circulation pattern and its effect on the mixing and dilution of waste discharges into the lake. The mixing and dilution of tracers introduced at various positions in the model lake were also studied under these same conditions of zero wind stress and constant throughflow and with the superposition of the westerly wind.

A highly idealized model of Lake Erie was also constructed and operated. This model was rectangular in shape, of constant depth, and had the same length to width ratio as Lake Erie. Circulation patterns, the decay of seiches, and mixing of tracers were also studied in this idealized model. The results obtained from the idealized rectangular basin helped to delineate the major effects of geometrical variations that were present in the scaled Lake Erie model.

This report recounts the development of the laboratory and the model lake and summarizes the analytical and experimental work accomplished thus far.

CIVIL ENGINEERING REPORT NO. 18.1
FACULTY OF ENGINEERING AND APPLIED SCIENCES
STATE UNIVERSITY OF NEW YORK AT BUFFALO

DYNAMIC MODEL STUDY OF LAKE ERIE

PART I. SIMILITUDE CRITERIA AND EXPERIMENTAL SET-UP

by

Ralph R. Rumer, Jr.

AUGUST 1970

Prepared under the Support of the

Federal Water Pollution Control Administration

Dept. of the Interior (Grant No. WP-00837)

Acknowledgments

The author would like to express his appreciation to Robert L. Ketter, George C. Lee, Lawrence Larkin, Richard Cipolla, and Robert Basehart who all had a part in the design of the rotating laboratory facility. A debt of gratitude goes to John Sarvey who almost singlehandedly erected, assembled, and put into operation the rotating laboratory. The many helpful suggestions made by Donald Harleman, Simon Ince, and John Hoopes are gratefully acknowledged. Thanks also go to Sherwood Prawel, Chee Tung, Ralph Cheng, C. C. Chien, and Mona Conner for their contribution to the investigation.

Very special thanks go to Lance Robson, Vedula Sarma, J. C. Shiau and Peter Buechi. Each played an important part in the development of the experimental arrangements and collection of data.

This work was supported by Grant No. WP-00837 from the Federal Water Pollution Control Administration, Department of Interior.

DYNAMIC MODEL STUDY OF LAKE ERIE

Part I. Similitude Criteria and Experimental Set-up

Contents

		page
I.	Introduction	1
II.	Similitude Requirements	4
	II.1 Equations of Motion	4
	II.2 Material Balance Equation	7
	II.3 Discussion of Physical Forces	10
	II.3.1 Geophysical Effects	10
	II.3.2 Meteorological Effects	11
	II.3.3 Temperature Effects	12
	II.3.4 Hydrological Effects	13
	II.3.5 Frictional Effects	13
	II.4 Model Laws	16
	II.5 Model Dimensions	19
III.	Design and Construction of Rotating Laboratory and the Model Lake	21
	III.1 Rotating Laboratory	21
	III.1.1 General Layout	21
	III.1.2 Materials and Type of Construction	21
	III.1.3 Structural Design	22
	III.1.4 Support Bearings	23
	III.1.5 Rotating Laboratory Drive Train	24
	III.1.6 Electrical Power Supply to Laboratory	25
	III.2 Model Construction	25

		page
IV.	Model Operation	27
	IV.1 Basic Procedure	27
	IV.2 Circulation Studies Without Wind	29
	IV.3 Detention Time and Dilution Factor Studies	32
	IV.4 Seiche Studies	35
	IV.5 Circulation Studies with Wind	36
V.	Summary	40
VI.	References	

I. INTRODUCTION

Physical rotating hydraulic models of flow systems in which the effect of the earth's rotation is important have been constructed and operated. The size of these models has varied from small table-top set-ups designed to simulate major atmospheric movements to massive turntables 46 feet in diameter designed for study of tidal estuary models. More recently, Harleman et al (1964) conducted a small-scale rotating model study of Lake Michigan. Although the model was highly idealized, significant information relating to wind-driven surface circulation and thermocline development was obtained. Chabert-D'Hieres and Hyacinthe (1967) constructed a small scale rotating hydraulic model to simulate large scale diffusion. The model was calibrated by carrying out a number of medium scale tests in the prototype and the model. Hoopes et al (1969) have constructed and operated a rotating model of Lake Superior. Their study concentrated on the wind-driven circulation for a homogeneous and a stratified lake condition.

Relatively speaking, there have been very few rotating hydraulic models constructed and operated. One obvious reason is the difficulty in making observations and monitoring the physical variables in the model while it is rotating. Another reason relates to the lack of knowledge and experience in operating such models. The distortion ratio (vertical length scale divided by horizontal length scale) is, by necessity, uncommonly large for these models and the degree of correspondence between such models and their prototypes is subject to question.

In spite of these considerations, interest in more detailed information of large scale fluid systems in which the effect of the earth's rotation is important has risen with the increasing concern over the disposal of wastes into the natural environment. It is anticipated that as the theory and practice of rotating hydraulic models becomes more sophisticated, there will be more usage made of this experimental technique for acquiring important information concerning complex flow systems. Such models will also be extremely useful in the calibration and verification of mathematical models. The ability to adequately accomplish this calibration in the prototype is limited because of the continually changing conditions in nature and the areal extent over which observations must be made. The importance of the problems to be studied calls for the continued exploration of the usefulness of rotating hydraulic models of large flow systems.

This report concerns a dynamic model study of Lake Erie being conducted at the State University of New York at Buffalo. The scope of the study includes both analytical and experimental investigations so as to provide basic information concerning water movement in the Lake. The knowledge and experience acquired in the course of this study should improve our capability to construct and operate rotating hydraulic models. This information should also assist in the overall water quality management of the Lake. The study complements recent field work and will aid in the interpretation and understanding of these field data. From the beginning, the objective of this investigation has been to develop in a systematic way an understanding of the response of the model Lake to various external factors. These external factors have been introduced individually at first. As knowledge and understanding were achieved

for these simple modeling situations, the more complex studies of combinations of these external factors were attempted. The studies reported here deal with the idealized conditions of zero wind stress (including the inflow of the Detroit River and the outflow of the Niagara River) and, under this same flow condition, with the superposition of a uniform westerly wind over the surface of the Lake. The water mass is isothermal and variations in density are assumed absent. Under these conditions, investigations of the circulation patterns were conducted. Also studied was the oscillation of the entire water mass in the model lake. In particular, attention was given to the transient effect that this mass oscillation had on the otherwise steady-state circulation pattern and its effect on the mixing and dilution of waste discharges into the lake. The mixing and dilution of tracers introduced at various positions in the model lake were also studied under these same conditions of zero wind stress and constant throughflow and with the superposition of the westerly wind.

A highly idealized model of Lake Erie was also constructed and operated. This model was rectangular in shape, of constant depth, and had the same length to width ratio as Lake Erie. Circulation patterns, the decay of seiches, and mixing of tracers were also studied in this idealized model. The results obtained from the idealized rectangular basin helped to delineate the major effects of geometrical variations that were present in the scaled Lake Erie model.

This hydraulic modeling project began in the Fall of 1965. The first stage included the design and construction of a suitable laboratory in which the model studies could be conducted (see Chapter III). Towards

the completion of this first stage the construction of the model of Lake Erie was begun. By the beginning of the year 1967 the laboratory and model were ready for use. This report recounts the development of the laboratory and the model lake and summarizes the analytical and experimental work accomplished thus far.

II. SIMILITUDE REQUIREMENTS

II.1 Equations of Motion

Earlier works of von Arx (1954) and Harleman et al (1962) have discussed the feasibility of modeling large water masses in which the effect of the earth's rotation was important. The basic criterion for achieving dynamic similarity between a model and prototype requires that the two geometrically similar bodies follow similar motion patterns under the influence of similar accelerating forces. For the present study, the important forces include the Coriolis force, gravitational attraction, and frictional resistance. Since, in Lake Erie, the ratio of horizontal dimensions to vertical dimensions is very large, a smaller scale reduction must be made for vertical distances than for horizontal distances. Such a model is termed "distorted" and is not uncommon in hydraulic models of rivers and estuaries.

The theoretical bases for the design of a hydraulic model of Lake Erie are the dynamical equations of motion for a fluid on a rotating planet. Using a left-handed rotating coordinate system with the x-axis directed towards the east, the y-axis towards the north, and the z-axis towards the earth's center (see Fig. 1) the equations of motion for an incompressible fluid can be written in vector form as

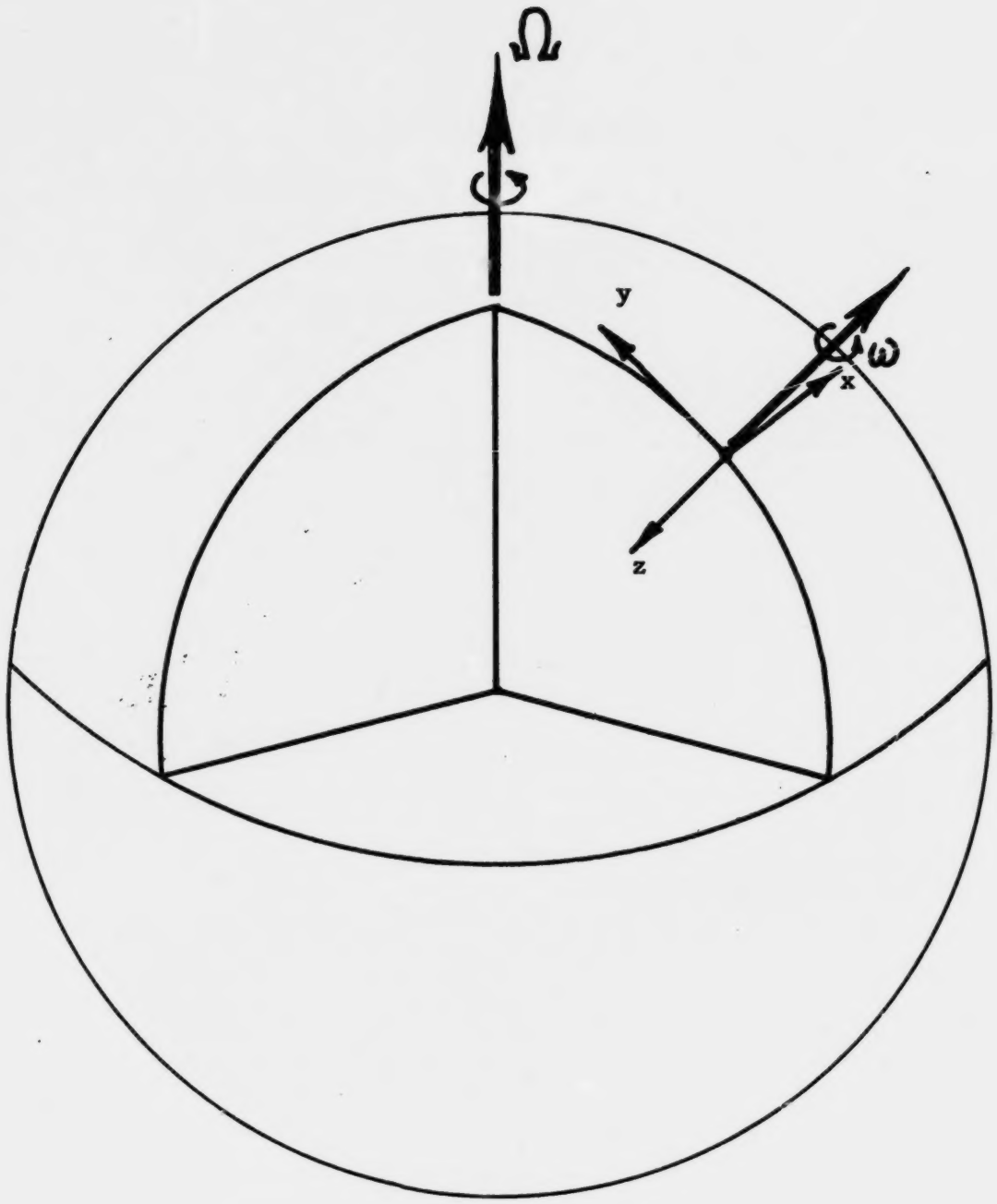


Fig. 1 Schematic Sketch of Coordinate System on Rotating Earth

$$\frac{\partial \vec{q}}{\partial t} + (\vec{q}\cdot\nabla)\vec{q} = -2\vec{\omega} \times \vec{q} - \frac{1}{\rho}\nabla p + \vec{g} + \epsilon \nabla^2 \vec{q} \qquad (2\text{-}1)$$

where

$\vec{q}$ = velocity vector = $\vec{i}\,u + \vec{j}\,v + \vec{k}\,w$ (u,v,w, are velocities in x,y,z coordinate directions)

p = pressure

ρ = mass density of the fluid

ϵ = kinematic eddy viscosity of the fluid (assumed here to be a constant)

$\vec{g}$ = acceleration of gravity = $\vec{k}g$

t = time

$\vec{\omega} = \vec{k}\,\Omega \sin \phi$

Ω = angular velocity of the earth (= $.73 \times 10^{-4}$ rad/sec.)

ϕ = latitude

∇ = vector operator = $\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}$

∇^2 = scalar operator = $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

$\vec{i},\vec{j},\vec{k}$ = unit vectors in the x,y,z directions

Introducing a reference velocity, U_o, and a reference length, L_o, the following dimensionless variables can be defined:

$$\vec{q}' = \frac{\vec{q}}{U_o}\,,\quad p' = \frac{p}{\rho U_o^2}\,,\quad t' = \frac{t}{L_o/U_o}\,,\quad \nabla' = \nabla L_o,\quad (\nabla')^2 = \nabla^2 L_o^2 \qquad (2\text{-}2)$$

Substituting equations (2-2) into (2-1) and simplifying results in

$$\frac{\partial \vec{q}'}{\partial t'} + (\vec{q}'\cdot\nabla')\vec{q}' = \frac{1}{R_o}(2\vec{k} \times \vec{q}') - \nabla' p' + \frac{1}{F}\vec{k} + \frac{1}{R_t}(\nabla')^2\vec{q}' \qquad (2\text{-}3)$$

The dimensionless groups appearing in equations (2-3) are as follows:

$$\mathcal{R}_o = \frac{U_o}{(\Omega \sin\phi) L_o}$$

and is called the Rossby number. The Rossby number is considered to represent the ratio of inertia forces to Coriolis forces, the Coriolis force representing the deflecting force due to the earth's rotation.

$$\mathcal{F} = \frac{U_o}{\sqrt{gL_o}}$$

and is called a Froude number. The Froude number is considered to represent a ratio of inertia forces to gravity forces.

$$\mathcal{R}_t = \frac{U_o L_o}{\epsilon}$$

and is called a "turbulent" Reynolds number. The "turbulent" Reynolds number is considered to represent a ratio of inertia forces to turbulent shear stresses.

Based on this analysis of the equations of motions (Eq. 2-1), the condition for complete dynamic similarity between a geometrically similar model and prototype is equality of $\mathcal{R}_o$, $\mathcal{R}_t$, and $\mathcal{F}$ between the model and the prototype. Equation (2-1) is approximate insofar as the kinematic eddy viscosity is not a constant nor is it a fluid property. Rather, it depends upon the character of the flow and will vary continuously throughout the flow field. In addition, there are extended periods in the prototype when the density, ρ, can not be considered a constant (i.e., seasonal stratification).

For a particular point on the earth's surface in the Northern hemisphere the rotational velocity vector of the earth can be resolved into two components, one normal to the earth's surface and one tangential to the earth's surface. The important component is the normal component

(Ω sin ϕ) which produces the Coriolis acceleration on water particles in motion. The tangential component (Ω cos ϕ) produces a vertical acceleration which does not significantly alter the gravitational constant and can therefore be neglected.

The dimensionless parameters appearing in equation (2-3) [R_o, R_t, F] represent ratios of the forces which have been presumed to affect the water movement in Lake Erie. It should be noted, however, that the external forces such as wind stress and atmospheric pressure disturbances are not directly included in this equation. In the mathematical solution of simplified versions of equation (2-3) these external forces are included as boundary conditions. Unfortunately, precise similitude criteria do not exist for the appropriate scaling of wind velocities for model studies. The methods by which similarity is accomplished between model and prototype, based on the dynamical equations and also including the effects of the external forces, are discussed later in this report.

II.2 Material Balance Equation

Mixing and diffusion in Lake Erie are primarily a result of the large scale eddy patterns that are generated by the combination of inflows and wind-driven currents and subsequently modified by the effect of the earth's rotation and the geometric configuration of the basin. Stratification is important in that it modifies these effects on the general mixing processes in the Lake. Mass oscillations will also have their effect upon mixing. It is expected that at least the large scale mixing processes will be similar in model and prototype. This would permit tracing pollutants that are discharged into the lake at various points on the perimeter, or from sites within the lake. The smaller scale turbulent mixing resulting from surface waves and bottom roughness

will not be modeled.

Consider an idealized lake basin of constant depth in which the flow can be assumed to be two-dimensional and parallel to the bottom of the basin. The basic equation for the material balance of a conservative substance that is being convected and dispersed by the flow in the basin is given by

$$\frac{\partial s}{\partial t} + u \frac{\partial s}{\partial x} + v \frac{\partial s}{\partial y} = E_{xy} \left(\frac{\partial^2 s}{\partial x^2} + \frac{\partial^2 s}{\partial y^2} \right) + E_z \frac{\partial^2 s}{\partial z^2} \qquad (2\text{-}4)$$

where s is the concentration of the conservative substance; u, v, represent the components of velocity in the x and y directions, E_{xy} is the mass transport coefficient due to turbulent diffusion and dispersion in the horizontal x-y plane, and E_z is the mass transport coefficient in the vertical direction.

Equation (2-4) can be put into dimensionless form by introduction of the following characteristic quantities:

$$u' = \frac{u}{U} \; , \; v' = \frac{v}{U} \; , \; x' = \frac{x}{L} \; , \; y' = \frac{y}{L} \; , \; z' = \frac{z}{Y} \; , \; s' = \frac{s}{S_o} \; , \; t' = \frac{t}{L/U} \qquad (2\text{-}5)$$

Inserting the new dimensionless primed variables into equation (2-4) gives

$$\frac{\partial s'}{\partial t'} + u' \frac{\partial s'}{\partial x'} + v' \frac{\partial s'}{\partial y'} = \frac{E_{xy}}{UL} \left(\frac{\partial^2 s'}{\partial x'^2} + \frac{\partial^2 s'}{\partial y'^2} \right) + \frac{E_z L}{UY^2} \left(\frac{\partial^2 s'}{\partial z'^2} \right) \qquad (2\text{-}6)$$

For this idealized situation, similitude between a model and prototype would require that

$$[E_{xy}]_r = Y_r^{\frac{1}{2}} L_r \qquad (2\text{-}7)$$

and

$$[E_z]_r = Y_r^{5/2} L_r^{-1} \qquad (2\text{-}8)$$

where the subscript denotes the ratio of the quantity in the model to that in the prototype. Y_r is the vertical scale ratio and L_r is the horizontal scale ratio. The ability to satisfy both of these equations will be quite limited. Measurements of E_{xy} and E_z will be needed from the prototype. The appropriate choice of Y_r and L_r in constructing a model might make it possible to simulate either E_{xy} or E_z in the model. Rarely will it be possible to simulate both according to equations (2-7) and (2-8).

Equations (2-7) and (2-8) when combined indicate that the ratio of horizontal diffusion to vertical diffusion in model and prototype is given by

$$\left(\frac{E_{xy}}{E_z}\right)_m = \left(\frac{L_r}{Y_r}\right)^2 \left(\frac{E_{xy}}{E_z}\right)_p \qquad (2\text{-}9)$$

From Csanady's work (1964), the ratio of horizontal diffusion to vertical diffusion in Lake Erie is on the order of 1000. This is highly variable depending on local currents and degree of stratification. Nevertheless, it does provide an order of magnitude for further consideration of mixing in a model. Since the flow in the model may very likely be laminar, it appears unlikely that correspondence between small scale mixing in the model and prototype will be obtained. This point needs further study, taking into consideration the approach used by Taylor (1953) for the dispersion of matter in laminar flow. It is possible that, by adapting Taylor's approach to the depth-averaged equation (2-4), more useful criteria for similarity of mixing can be obtained. This possibility will receive further attention.

It has been known for some time that as the size of a diffusing stream becomes larger, then larger and larger scales of turbulence contribute to the mixing. Apparently the magnitude of the mass transport

coefficient increases greatly with the size of the area being considered. One expression for this dependence of the mass transport coefficient (or eddy diffusivity) on the scale of the system is given by Richardson's law (Orlob, 1959) as:

$$E_{xy} = .01\ L^{4/3} \qquad (2\text{-}9)$$

where

L = scale of the system in (cm.)

E_{xy} = mass transport coefficient in $\left(\frac{cm^2}{sec.}\right)$

Brooks (1960) has considered the spreading of a substance in an ocean current, taking into account this dependence of the mass transport coefficient on the scale of the system. Taking the size of the western basin of Lake Erie as a limiting scale of the system, a value of E_{xy} on the order of $10^6 cm^2/sec.$ is obtained. A plume 1 mile in width would give a value of $E_{xy} = 8.7 \times 10^4\ cm^2/sec.$ Based on prototype field studies Csanady cites a range of $700 < E_{xy} < 4400\ cm^2/sec$ as typical for the western basin of Lake Erie. His plume widths were of the order of 100 m which according to equation (2-9) would give $E = 2200\ cm^2/sec.$, a value within the range measured.

II.3 Discussion of Physical Forces

II.3.1 Geophysical Effects

For typical water surface slopes experienced in large lakes and for typical water velocities, it can be shown that the Coriolis force per unit mass of water and the gravitational force per unit mass are comparable in magnitude. These water slopes and surface currents are primarily generated by gravitationally induced throughflows and by wind stress. The size of thewater mass involved in Lake Erie plus the

relatively slow motions associated with the movement of this water mass point toward the importance of the effect of the earth's rotation upon this motion as viewed from the rotating earth itself. The variation of the magnitude of this Coriolis force due to changes in latitudinal position on the earth's surface is normally included in the analysis of ocean currents and atmospheric movements. In the case of Lake Erie this variation is very small and can be neglected. Thus, an average value for the latitude of Lake Erie can be used in calculating the magnitude of the Coriolis force at any position in the lake.

II.3.2 Meteorological Effects

The action of wind stress is one of the more important of the physical forces affecting the circulation in Lake Erie and must be incorporated in the model study. Winds and relatively rapid moving pressure disturbances are the primary causes of seiche motion. Seiche motion will undoubtedly have an important transient effect on circulation patterns and the mixing and dilution of waste discharges into the lake. The ability to demonstrate whether this aspect can be suitably modeled and observed in the proposed model was an important objective of this study. The exact character of these seiches in basins of variable geometry and conditions for their occurrence are not fully understood, and the model study should provide basic information on this subject. The procedure followed in scaling wind velocities over the model lake is discussed in Chapter IV.

II.3.3 Temperature Effects

Seasonal modifications of the circulation patterns and seiche motions result from density stratification due to vertical thermal (temperature) differences. This seasonal stratification has been correlated with observed deficiencies in dissolved oxygen in the bottom waters of the Lake [Carr (1962), Carr et al (1963)]. An important difference between the seasonal stratification in a shallow lake such as Lake Erie and a deeper lake such as Lake Michigan is that, whereas in the deeper lake the thickness of the upper layer is small compared to the total depth, in the shallow lake the upper layer represents a large percentage of the total depth. The modification of seiche action and associated internal waves during the seasonal stratified condition of the Lake is probably not as pronounced as in the deeper lakes such as Lake Michigan.

It has already been demonstrated that the seasonal stratification can be simulated in model lakes by thermal devices (lamps) or artificially by liquids of different densities [Chapell (1963), Harleman et al (1964)]. The behavior of the two-layered system is well known to be different from the isothermal system under the same external conditions. Harleman et al (1964) discovered that the wind-driven circulation pattern in a small scale rotating model of Lake Michigan varied with the thickness of the epilimnion. It is to be expected that changes in the gross circulation pattern and in the residence times of contaminants moving through the lake will occur when the lake is stratified.

II.3.4 Hydrological Effects

The average outflow from Lake Erie through the Niagara River is approximately 219,000 cubic feet per second (cfs). Annually, this amounts to almost 40% of the Lake volume. Similarly, the inflow from the Detroit River averages about 175,000 cfs or 32% of the Lake volume annually. The difference is made up by the remaining tributaries and precipitation onto the Lake. For Lake Erie, these inflow-outflow quantities are significant and must be reproduced in a model study. Other tributaries are of much less importance and need not be considered in the initial phases of the model study.

Lake Erie is, on occasion, subject to complete ice cover during the winter months. The location and identification of relatively motionless areas of the Lake, and therefore possible areas of poorer water quality, under the conditions of ice cover would be very useful. The elimination of surface wind stress during the periods of complete ice cover indicates that the influence of the inflow at Detroit and the outflow at Niagara should be greatest at that time.

II.3.5 Frictional Effects

The shallowness of Lake Erie indicates that bottom frictional effects may be important in influencing the motions in the Lake. Platzman and Rao (1963) discuss the possible damping influence of bottom friction on seiche action. As will be discussed later, frictional effects cannot be directly scaled in a distorted model. Past experience with river and estuary models has shown that the effects of bottom friction can be incorporated in a distorted model. The procedure is one of trial and error and usually requires addition of roughness elements to increase

the resistance of the model boundaries. Unfortunately, the systematic and detailed field observations usually obtained for verification of distorted river and estuary models are not readily available in the case of Lake Erie. For this reason, the process of model verification is not exact and confidence in the model is obtained gradually. In unidirectional flows such as river models, these roughness elements are usually rectangular strips. In a large lake model where two-dimensional motion predominates, the roughness elements would have to be of some shape such that the drag exerted by the strip would be more or less independent of current direction (e.g. cylinders instead of strips).

Although approximate predictions as to the nature of this artificial roughness can be made using the uniform flow equations of Chezy or Manning, ultimately adjustments will have to be made in the size, number, and concentration of the elements such that certain times of travel in the model agree with the prototype. It is possible that prototype information on the damping of longitudinal seiches will be helpful in the adjustment process of frictional elements. This point is discussed further in Section VIII.

It can be shown that for unidirectional flow in wide shallow channels the ratio of the Manning roughness coefficient in a distorted model to that in a prototype is given by

$$n_r = \frac{Y_r^{2/3}}{L_r^{1/2}} \qquad (2\text{-}10)$$

where n_r is the ratio of the Manning roughness coefficient in model and prototype, Y_r is the vertical scale ratio in model and prototype, and

L_r is the horizontal scale ratio in model and prototype. For large distortion ratios (Y_r/L_r), eq. (2-10) requires that the model have a roughness coefficient very much greater than the prototype. In the derivation of equation (2-10) a basic assumption is that the flow is uniform and unidirectional. This assumption is probably not applicable in the case of the complex flow patterns to be expected in Lake Erie. For this reason it would be unreasonable to arbitrarily roughen the model in accordance with Eq. (2-10).

Another approach to determining the frictional resistance needed in the model is through the use of the so-called Ekman depth of frictional influence, D, where

$$D = \pi\sqrt{\frac{\epsilon_z}{\Omega \sin \phi}} \tag{2-11}$$

In this equation ϵ_z is the momentum transport coefficient in the vertical direction and $\Omega \sin \phi$ is the magnitude of the rotation vector at the position of interest. By use of this equation in normalized form and letting the ratio of the depths, D, in model and prototype be given by the vertical scale ratio, one obtains

$$\epsilon_{z_r} = \frac{Y_r}{L_r}^{5/2} \tag{2-12}$$

where ϵ_{z_r} is the ratio of the momentum transport coefficient in model and prototype. [Note the agreement between eq. (2-12) and eq. (2-8).] Typically Eq. (2-12) will specify momentum transport coefficients to be smaller in the model than in the prototype. This is certainly to be expected in a small-scale model of Lake Erie. The ability to satisfy eq. (2-12) will depend on the value of ϵ_z chosen for the prototype and the nature of the flow in the model (whether laminar or turbulent).

The use of equation (2-12) as an approach to the problem of scaling friction would seem to have some validity. Further discussion on this point can be found in Chapt. II.4.

During periods of seasonal stratification, the influence of bottom friction would be at a minimum. If the roughness elements were of such a height that they protruded through the cool bottom layer into the warmer epilimnion, excessive vertical mixing would result in the model which would destroy the two-layered system.

In summarizing the discussion of the physical forces the following points are made: (1) a hydraulic model of a large lake such as one of the Laurentian Great Lakes will have to be rotated so as to include the effect of the earth's rotation, (2) the effect of wind stress on motion in the lake is significant and should be incorporated into the model study, (3) the inflow of the Detroit River and the outflow of the Niagara River have a significant effect on the water movement in Lake Erie and should be incorporated into the model study, (4) the seasonal stratification of the lake causes significant alterations in the behavior of the lake mass and should be incorporated into the model study if possible, (5) the effect of friction is important in shallow basins and attention should be given to the verification of the model by whatever means appropriate.

II.4 Model Laws

The ratio of mean depth to length for Lake Erie is of the order of 1 to 20,000. This geometric configuration of the prototype requires that the model be vertically distorted in order to keep the horizontal dimensions within reason and also to avoid water surface tension effects

that would otherwise occur with very small depths. The use of a distorted model assumes that the water motions are predominantly horizontal. The vertical motions that occur in a large lake such as upwelling and seasonal overturning will still occur in the model lake but direct comparison of such motions with the prototype will not be possible.

Gravitational similitude is obtained by equality of the Froude number, F, in model and prototype. Its importance has already been alluded to in relation to mass oscillations, water surface slopes generated by wind stress or pressure disturbances, and thermally induced density variations. In the following development, the subscript "p" will denote prototype quantities, "m" will denote model quantities, and "r" will denote the ratio $\left(\frac{\text{model}}{\text{prototype}}\right)$ of the same quantity in model and prototype.

The velocity ratio for a distorted Froude model is

$$U_r = Y_r^{\frac{1}{2}} \tag{2-13}$$

where Y_r is the vertical scale ratio, Y_m/Y_p. The time ratio is derived from the rate of motion in the horizontal direction and is given by

$$T_r = \frac{L_r}{Y_r^{\frac{1}{2}}} \tag{2-14}$$

where L_r is the horizontal scale ratio L_m/L_p.

Rotational similitude in model and prototype is achieved by equality of the Rossby number, R_o, in model and prototype. A model of Lake Erie would be rotated about its own center of gravity so as to simulate the normal component of the earth's rotation vector at the mean geographic latitude of the Lake. Equating the Rossby number in model and prototype yields

$$\omega_r = \frac{U_r}{L_r} \tag{2-15}$$

Since $\omega_r = (\Omega \sin \phi)_r$ and $U_r = Y_r^{\frac{1}{2}}$, equation (2-15) gives

$$\omega_m = \frac{(\Omega \sin \phi)_p Y_r^{\frac{1}{2}}}{L_r} \tag{2-16}$$

Equation (2-16) becomes, using $\Omega = .73 \times 10^{-4}$ rad/sec and $\phi = 42^\circ$,

$$\omega_m = \frac{(0.5 \times 10^{-4}) Y_r^{\frac{1}{2}}}{L_r} \tag{2-17}$$

Ratios for determining other model quantities such as inflow and outflow can be determined as follows:

$$Q_r = (\text{Area})_r \, (\text{Velocity})_r \tag{2-18}$$

Since area is given by $L_r Y_r$ and the velocity ratio by $Y_r^{\frac{1}{2}}$, equation (2-18) becomes

$$Q_r = L_r Y_r^{3/2} \tag{2-19}$$

Frictional forces as represented by the "turbulent" Reynolds number R_t cannot at the present time be directly modeled by equating this third parameter in model and prototype. This is because the kinematic eddy viscosity, ϵ, is not entirely a fluid property and hence is not known a priori. It is dependent upon the motion itself. Frictional effects, when important, and the relationship between wind stresses and surface currents must all be incorporated by empirical procedures and through the process of model verification. In the case of wind stresses, one technique would be that the model wind velocities be adjusted until model surface currents agree with observations in the prototype for a prescribed set of conditions. An alternate procedure will be discussed in Chapt.IV.5.Since frictional effects would have to be included at the same time it is anticipated that the process of model verification will

be tedious and complex, and will be accomplished gradually during the operation of the model and as more prototype information becomes available.

The rotation of the model about its own center of gravity will give rise to centrifugal accelerations. These accelerations will produce a free surface in the model that is a paraboloid of revolution in accordance with the equation

$$y - y_o = \frac{\omega_m^2 r^2}{2g} \qquad (2\text{-}20)$$

where y_o is the water depth at the center of rotation of the model, y is the distance to the free surface of the model measured from a plane surface which is tangent to the model bottom at the center of rotation and r is the radial distance measured from the center of rotation. All vertical measurements in the model will have to be made with regard to this curved plane (see Fig. 2).

II.5 Model Dimensions

The choosing of scale ratios is to some degree an intuitive choice based on experience. It should be noted, however, that in the case of rotating vertically distorted Froude models of large bodies of water, there is little experience from which judgment can be made. One possible guide would be the recommendation of von Arx (1962). Ultimately, the model dimensions for the proposed Lake Erie model must take into account the space that is available in the laboratory.

In estimating model size the following average dimensions of the prototype Lake Erie were used:

Length = 1,270,000 ft (240 miles)

Width = 264,000 ft. (50 miles)

Depth = 64 ft.

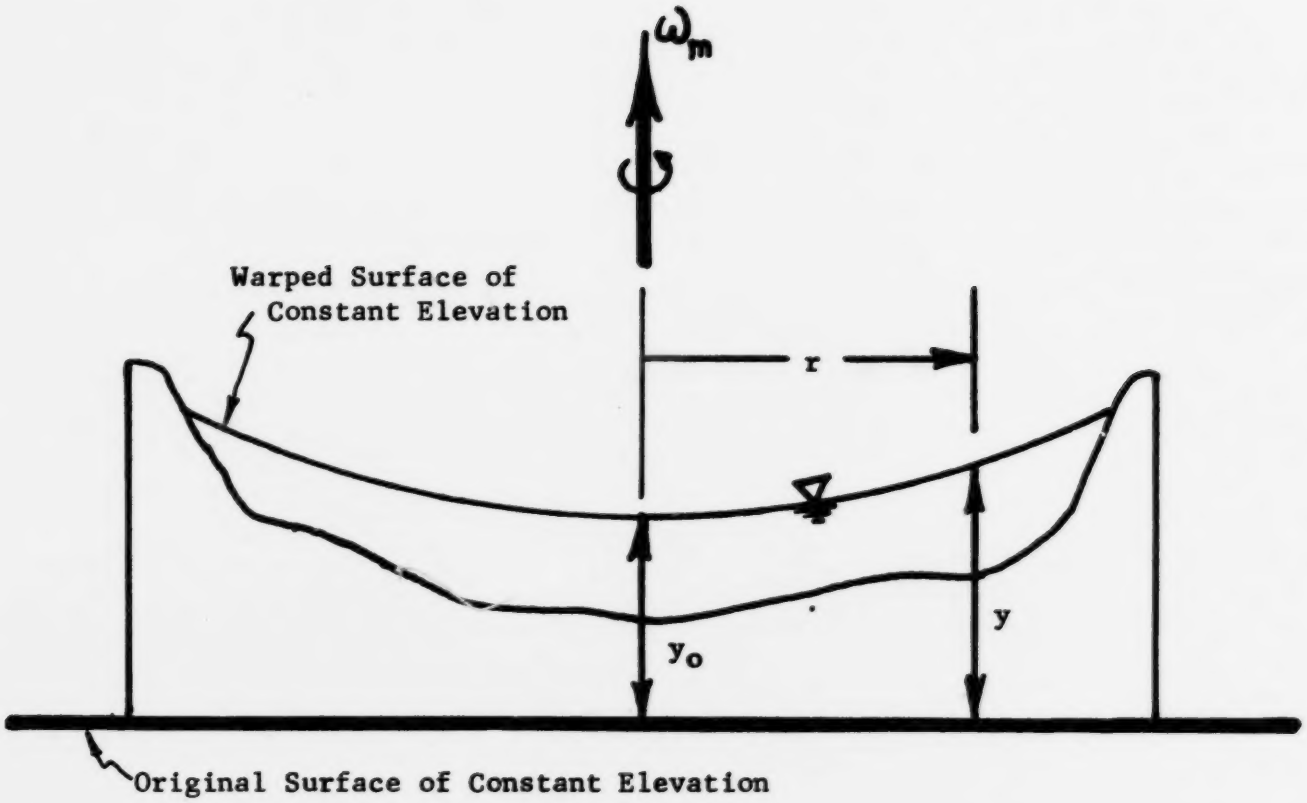


Fig. 2 Surface of Constant Elevation in a Rotating Model

In view of the lack of experience in this type of modeling it was felt unwise to attempt a large model until more is known about the degree to which correspondence can be obtained in model and prototype. With this in mind, the following scale ratios were chosen for the model lake.

$L_r = 1/200{,}000$

$Y_r = 1/500$

Thus, the model lake would be approximately six feet in length, an average of 1.33 feet in width, and have an average depth of slightly over 1½ inches. The maximum depth in Lake Erie is about 210 feet, and this would correspond to a maximum depth in the model of 5 inches. Thus, the distortion ratio (Y_r/L_r) is 400. The distortion ratio in the Lake Michigan model was 500 (Harleman et al, 1964).

The angular velocity of the model as given by equation (2-17) would be 0.437 rad/sec. This corresponds to about 4.17 revolutions per minute. Assuming a model length of 6.35 feet, the tangential velocity of the model extremities would be about 2.78 feet per second. The velocity ratio would be [see eq. (2-13)]$U_r = 1/22.4$ and the time scale would be [see eq. (2-14)] $T_r = 1/8930$. One prototype day would correspond to 9.68 seconds in the model. A prototype surface velocity of 0.5 feet per second would correspond to a velocity of approximately 0.68 cm/sec. in the model. The inflow at the Detroit River of 175,000 cfs would correspond to a model inflow of 2.21 cubic centimeters per second. The Niagara River outflow in the model would be 2.79 cubic centimeters per second, corresponding to a discharge of 219,000 cfs in the prototype. A mass oscillation in the prototype causing periodic variations in Lake level at one end or the other of 6 feet (commonly reported) would correspond to a variation in the model lake level of slightly more than

3.5 mm. The 14.1 hour period associated with the longitudinal seiche in the prototype would correspond to approximately 5.67 seconds in the model.

III. DESIGN AND CONSTRUCTION OF ROTATING LABORATORY AND THE MODEL LAKE

III.1 Rotating Laboratory

III.1.1 General Layout

As outlined previously, rotation of the model of Lake Erie is required in order to reproduce the Coriolis effects from the earth's rotation. This in turn required a complete rotating laboratory including a platform and exterior framing. The platform measuring 10 feet by 17 feet accomodates the lake model itself, instrumentation, and walkways and work space for technicians. The exterior framing provides inside headroom of 7 feet 7 inches and supports lights, heat lamps, fans and cameras. This rotating laboratory is completely enclosed to provide a chamber in which the environment can be carefully controlled. The laboratory is mounted with a thrust bearing at the bottom and a radial bearing at the top. The thrust bearing transfers the weight of the structural frame, model, equipment, and personnel to the foundation. Both bearings resist horizontal loads. The upper bearing is supported by two intersecting structural bents. See Fig. 3.

III.1.2 Materials and Type of Construction

The main consideration in the selection of a material for the structural framing was the need for adequate mass and stiffness to control the deflections and the vibration amplitudes. This factor, together with considerations of cost and availability, pointed to the selection of structural steel. All of the structural connections to the steel

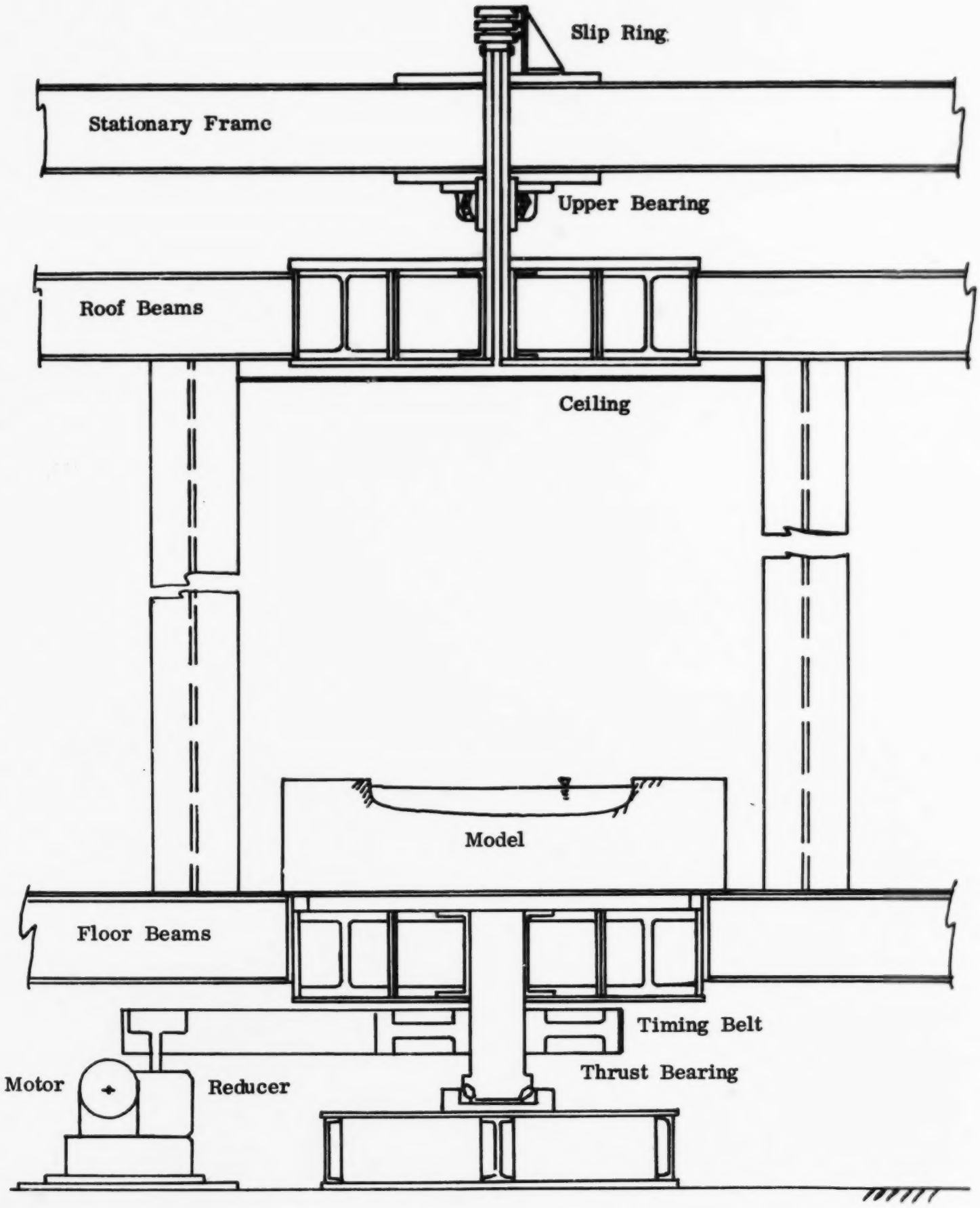


Fig. 3

Schematic Sketch of Rotating Laboratory

framework were bolted connections, using high-tension bolts. Although welding would have given a more rigid structure, bolted connections were used because of the possibility that the entire facility might eventually be relocated.

III.1.3 Structural Design

a. Rotating Laboratory Framing

The platform framework and the overhead or ceiling framework are gridworks of wide-flange steel beams with five longitudinal beams (four are continuous) interconnected by short, wide-flange beam diaphrams. Eight wide-flange steel columns, located at or near the ends of the four continuous beams, connect the platform framework to the ceiling framework. The critical requirement in the design of these wide-flange members in the framework was the restriction on the vertical deflection due to the movable loads on the platform. This deflection was limited to 1/64 inch, the expected precision of the vertical measurements in the lake model. The assumed critical loading situation occurs when the recorder (500 lbs.) and four technicians (800 lbs.)stands at one end of the platform.

b. Upper Bearing Support

The upper bearing is simply a radial bearing and carries the unavoidable lateral loads resulting from an imbalance of mass on the rotating laboratory and misalignment of the axis of rotation. This upper bearing is supported by two structural steel bents. The planes of the structural bents are perpendicular to each other and the horizontal members of the bents intersect at the location of the upper bearing. The bents were designed so that the expected lateral load,

estimated at 2000 lbs, produces a lateral displacement less than 1/32 inch.

c. Structural Footings

The total weight of the rotating laboratory, some 35,000 lbs, is transmitted through the lower bearing to a reinforced concrete foundation. To distribute this load with allowable pressure over the concrete foundation, the lower bearing rests directly on a steel grillage footing which in turn rests on and is anchored to the concrete foundation. The existing concrete slab at the site of the laboratory was not adequate to carry either this load or the reactions from the structural bents supporting the upper bearing. Thus, portions of the existing slab had to be removed. Since excavation was necessary for a new foundation, it was decided to place the lower bearing, its steel footing and the concrete foundation on which it bears, below the existing floor level along with the motor and drive mechanism for rotating the laboratory. This permitted the rotating laboratory to be located more conveniently nearer the existing floor level. This steel reinforced concrete foundation under the lower bearing was cast integrally with the steel reinforced concrete foundations for the structural bents supporting the upper bearing.

III.1.4 Support Bearings

The thrust bearing supporting the rotating laboratory must carry a thrust load of 35,000 lbs. and an unavoidable radial load not expected to exceed 2000 lbs. The maximum expected rotation rate is 5 rpm and the required bearing life is 10,000 hours. Several types of thrust bearings were considered, including an air cushion system. The bearing selected was a self-aligning spherical roller thrust bearing (SKF No. 29422). Similarly, the radial bearing stabilizing the laboratory

at the top must have a life of 10,000 hours at 5 rpm while carrying a radial load of 2000 lbs. The bearing selected for this use was a tapered roller bearing (Dodge Special Duty Flange Bearing with shaft size of 4 5/16").

III.1.5 Rotating Laboratory Drive Train

One of the main problems in building the proposed hydraulic laboratory was the requirement of an absolute minimum of vibration from any source, especially the rotational drive train. This consideration was uppermost in the selection of the drive train. Power requirements to rotate the laboratory are small in spite of the massiveness of the laboratory since the maximum speed is only some 5 rpm and since a fast starting time is neither necessary nor desirable for hydraulic modeling. For example, it would only take some 0.35 HP to bring the laboratory from rest to 5 rpm in one minute. The drive train selected from various manufacturer's recommendations is a variable speed motor (Dynamatic Model ACM 903, Range 0-1670 rpm) operating through a 75:1 speed reducer (Morse Model 50-DV) which in turn drives a timing belt sprocket (Morse No. 18-XXH-500) which is connected by a steel-wire reinforced rubber timing belt (Morse No. 1440-XXH-500) to a belt sprocket (Morse No. 72-XXH-500) which is mounted on the shaft of the rotating laboratory. The belt sprockets reduce the speed 4:1, giving a total reduction of 300:1 between the motor and the laboratory shaft. Thus the range of possible rotation rates for the laboratory are from 0 - 5.56 rpm.

III.1.6 Electrical Power Supply to Laboratory

Inside the rotating laboratory, electrical power is necessary for lighting, environmental control such as air-conditioning, heating lamps, and fans. It is also necessary for instrumentation and the hydraulic pumping system. To meet these needs, both 110 and 220 voltage were supplied to the rotating laboratory through a slip ring mounted on the rotating shaft above the top, radial bearing. The slip ring consists of three copper brushes in contact with three copper rings which are fastened to and separated by fiberglass disks.

III.2 Model Construction

The basic building material was 6" x 12" x 48" blocks of polyurethane foam (manufactured by National Gypsum under the trade name Zer-O-Cell). This material was selected because of its light weight and adaptability to machining.

Cross-sections of the lake were determined from U.S. Lake Survey Chart No. 3, Lake Erie, polyconic projection, water level of 568.6 ft. referred to the International Great Lakes Datum of 1955. For all practical purposes, it was considered that the chart was accurate to ± 6 feet in the prototype Lake Erie or approximately ± 1/8" in the model.

The cross-sections were adjusted according to Equation (2-20) so that the model depth when rotating would be a correct geometric representation of Lake Erie. At the extreme distance from the center of rotation this adjustment is approximately 0.9 inch and across any particular cross-section the adjustment can vary as much as 0.2 inch.

Computer programs were developed to make the appropriate conversions and adjustments from the Lake Survey Chart and produce a graphical cross-section on an associated Cal-Comp plotter.

Cutouts of these cross-sections were affixed to the polyurethane blocks and with the aid of a pantograph, contour lines were transferred from the Lake Survey chart to the polyurethane block. A milling machine with a standard steel tool was then used to trace out the contour lines, keying for depth on the cross sections (see Fig. 4).

After the individual blocks were milled they were all tied together with rods below the lake bed. One edge of the milled "plateaus" represented a finished surface elevation and by using wood sandpaper, it was possible to remove excess foam from finished edge to finished edge using the Lake Survey chart for relative curvature between contour lines. The contours were blended from block to block and then the blocks were cemented together with a two-component epoxy glue.

The entire lake bottom and upper block surface were then coated with a two-part epoxy resin filler having a pot life of 4 to 5 hours. This application left the lake surface extremely rough and it was then necessary to sand with varying grades of wood sandpaper to produce a smooth finish. The smooth surface was then painted with a two-part waterproof epoxy paint (2-3 hour setup time) resulting in a highly durable watertight model.

Because of the heavy labor in refinishing the epoxy resin filler and because of the loss of dimensional control due to the variable filler thickness, experiments were conducted using a simulated polyurethane lake painted with epoxy. The results appeared satisfactory, three coats of paint being sufficient to produce a watertight, smooth

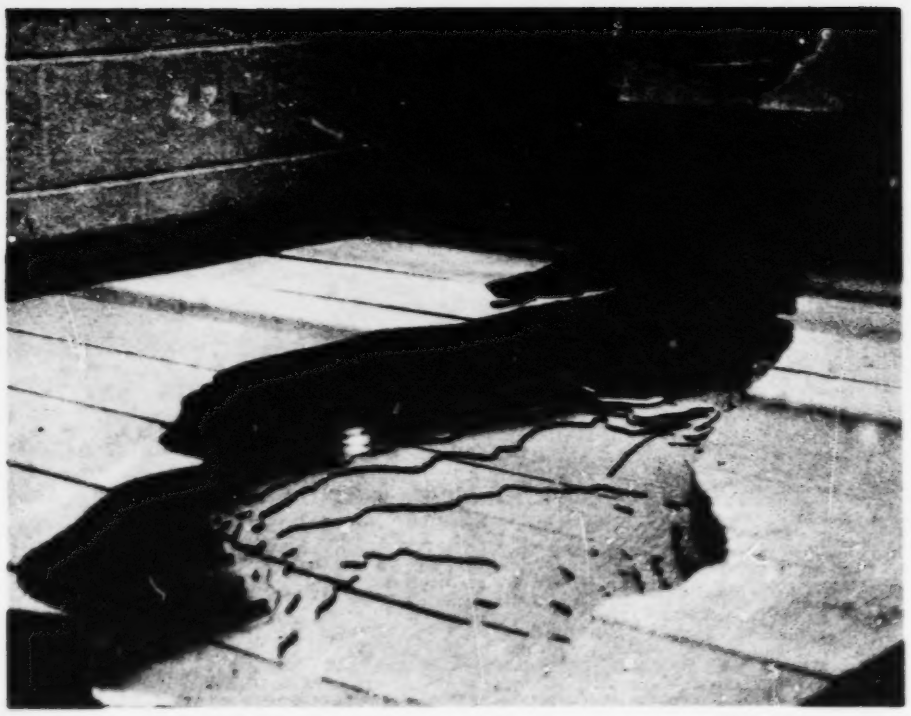


Fig. 4 Photograph of Lake Erie Model Under Construction

model. It is recommended that for future model construction the application of epoxy resin filler be eliminated.

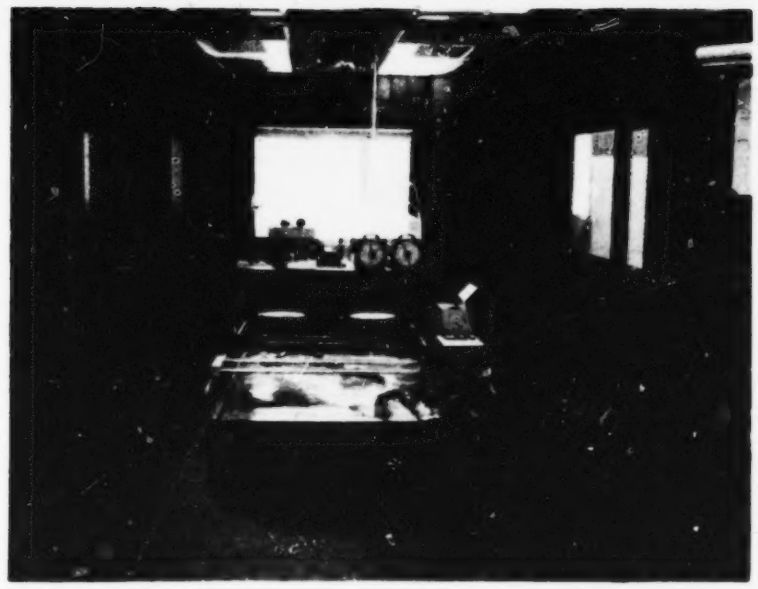
The entire grouping of sections was encased in a plywood frame so as to give extra strength and rigidity to the completed model lake. (See Fig. 5). The inflow and outflow to the model lake were accomodated by providing an outlet weir, return channel and sump. These were built into the polyurethane block system. The weir is of the adjustable V notch type, constructed of metal and epoxied to the model. For one experimental program it was necessary to epoxy polyurethane tubing to the model. This also worked satisfactorily.

IV MODEL OPERATION

IV.1 Basic Procedure

The rotating housing was provided with adjusting bolts so that the axis of rotation could be made vertical. These bolts are adjusted until the bubble in a precision level (one division = .0005"/ft) did not vary more than 1/2 a division while the room is rotating. This meant that the wave height due to the axis of rotation not being plumb would be less than .003 in. Once the axis had been plumbed, it was necessary to check it periodically and readjust as required. The housing will operate from one to two months without readjustment. The model was positioned in the room so that the center of rotation used for calculating cross-sections was directly on the axis of rotation. The axis of rotation could be determined by positioning a transit outside the room and moving a point around the ceiling until no motion was observed as the room rotated. Observations were made through doorways and other open areas, and not through glass. The model center was located by string lining

Outside View of Rotating Laboratory



Interior View of Rotating Laboratory Showing
Lake Erie Model and Instrumentation

Fig. 5 Photographs of Completed Lake Erie Model and Laboratory

between easily recognizable points on the shoreline. This positioned the model within an inch of the desired location, resulting in a maximum vertical error of 0.04", well within the construction accuracy of 0.12".

The model was initially leveled in the rotating housing to produce desired depths at a few key places with the correct volume of water. Marks were then made along the side of the model frame so that after any subsequent moving the model could be returned to the same position.

Based on cross-sections of Lake Erie, the model volume of water was calculated to be 24.9 liters. Using model scale ratios this corresponds to 17.6×10^{12} cf in Lake Erie. A volume of 17×10^{12} cf has been reported [FWPCA Report, 1968]. A tank was calibrated for the calculated water volume, filled the day before use, and placed in the laboratory so that the water temperature and ambient room temperature would be the same. This procedure was later simplified by filling the model lake with water a day prior to experiments.

To achieve steady state inflow and outflow without changing the volume, an additional amount of water was put in the recirculation sump and the depth marked. After the laboratory had achieved the correct rotation rate the throughflow was superimposed. The outlet weir was then adjusted until there was no change in sump depth for a 10-minute period. Experiments have indicated that within the limits of experimental accuracy, the time from start of rotation to steady state is less than 10 minutes.

As another measure of the time to reach steady state, Liggett (1969) gives the spin-up time (the characteristic time for a fluid body to establish steady, rotational motion) as

$$t = \frac{H}{\sqrt{\nu\omega}} \qquad (4 - 1)$$

where H is the depth, ν the kinematic viscosity, and ω the angular velocity. Using a maximum depth of 5" the Lake Erie model gives a spin-up time of 3.4 minutes. This time is less than the observed time to achieve steady state but of the same order of magnitude.

The sump is 16" x 6" so that control of sump levels to $\pm$ 1 inch provides volume control to $\pm$ 7% and lake level control to $\pm$ 0.01". For some experiments approximately .01% of a surface active agent (Zonyl A, manufactured by Dupont) was added and mixed in the measuring tank to reduce surface tension. This product performed excellently, reducing the surface tention by 70%. Used in these quantities, foaming was negligible and the water appeared only slightly cloudy. Flow regulation and control was accomplished using a constant head device with controlling valve; full flow meter (Fischer and Porter Co., Hatboro, Pa. - flowrator tube 01N 150 A) with a range from 10 to 400 cc/min; and a centrifugal vertical immersion pump (Little Grant Pump Co., Oklahoma City, Oklahoma - model CP-500) capable of 9500 cc/min at 1 foot of head. During the operation of the model this system was checked periodically and did not vary more than $\pm$ 5 cc/min. Assuming the system to be left unattended for an hour, this would produce errors of $\pm$ 0.02 inch in water surface elevation and $\pm$ 11% in volume control. Actually, these should be considered as extreme conditions since the system is usually quite steady and reliable.

IV.2 Circulation Studies Without Wind

This experimental phase has proceeded in a variety of ways. Attempts have been made to describe the general circulation under steady inflow and outflow using surface drogues, neutrally buoyant dyes and Rhodamine B.

The primary surface tracer was Aluminum Bronze powder (Ohio Bronze Powder Co., Cleveland, Ohio) of unknown density. By carefully sprinkling this powder on the water, it was possible to observe particles of the order of .001 inch. Two types of observations could then be performed: 1) cover the surface with many particles so that extensive streamline development could be observed, 2) track individual particles to determine velocity magnitudes and direction. The full surface covering is useful to determine qualitatively the circulation, either by motion picture or streak photography. Individual particle observation is useful in developing quantitative information on the flow at specific points.

Motion pictures were made using a Bolex H16 Reflex Camera nominally set at 16 frames/sec. The camera was driven with an electrical motor so that hand cranking was not necessary and the camera went at constant speed. The camera was calibrated at 16 fps but because projection rates may vary, a timing clock was included in the picture. It was hoped that synoptic model pictures of the particle motion could be made, using a Kinoptic 57 mm focal length lens. This lens gives from the lens face an 87° field of view on the frame horizontal. Unfortunately, resolution of fine aluminum bronze particles is very poor so that this lens could only be used for large scale observations such as dye spreading. Since the wide angle lens was unsatisfactory, surface circulation studies were made of smaller areas, using an Agenieux zoom lens.

To determine surface velocity vectors at individual points on the model, a clear plastic square was inscribed with 1" and ½" diameter circles graduated in 10° increments. It was possible to position this standard within 1/4" of the water over the major flow paths and track

individual particles, noting their incoming and outgoing azimuths and the time to traverse the known distance. For most of the measurements the larger circle was used and it was only for velocities less than about .127 cm/sec. that the half inch diameter circle was used. The maximum error in time observation is estimated at one second giving for a typical velocity of 0.1 ips, a 10% error. For the same typical case a standard deviation for the velocity was observed as 6% of the mean, both figures comparing well with each other.

During the course of an experiment the room doors were shut and ceiling air conditioners turned on. To determine if the air circulation pattern in the laboratory was influencing the surface circulation, velocity measurements were made at a few selected points with the air conditioners turned off. There was no discernable effect.

The question was raised that since the density of aluminum bronze powder particles is greater than water and they are supported by surface tension forces, they may behave differently than the water particles do. In particular, the tracers may be affected more by the generated centrifugal forces, and therefore may move in a surface film differently from neighboring water particles. If the centrifugal force query were true, then the East-West motion of particles west of the center of rotation would be retarded and that of particles east of the center of rotation would be increased. Tracers less dense than water will be supported primarily by buoyant forces and hence are not dependent on the surface tension film. If this query were important, they would behave differently. To test both these questions, paraffin type tracers with the same density as sea ice were tested in several places. The results compared favorably with those obtained using Aluminum Bronze powder so that these

may be considered satisfactory for determining surface currents.

Dye with a specific gravity of 1.000 was made using Merian fluid concentrate (No. D-2930, the Merian Instrument Co., Cleveland, Ohio). This dye was injected into the water and the resulting motion observed. By rapidly injecting dye across the lake, a continuous line is formed at time approximately zero. Observations were then made as a form of velocity profile developed. Since injection with a hypodermic needle gives the fluid an initial downward momentum, the dye distributes itself over the entire depth and initially disturbs the model flow pattern. The lake flow pattern appears to be reestablished within seconds so that observations after 15 seconds are considered representative of the model behavior. This approach was used qualitatively to determine an average (over the depth) general circulation which in some cases is different than the surface circulation observed with aluminum bronze drogues.

Large quantities of neutrally buoyant dye were added to the system at the sump to observe and photograph the predominant flow path through the model lake.

IV.3 Detention Time and Dilution Factor Studies

The primary method of conducting detention time studies has been to inject a tracer in the Detroit River and obtain a concentration history plot as water flows out the Niagara River (recovery curve). Initially, dilute Sodium Chloride solutions were used and changes in conductivity detected with a conductivity bridge (Industrial Instruments Inc. model RC 16B2). Samples of the outflow were obtained with eyedropper type conductivity cells at regular intervals. Results with this technique were erratic and a peak conductivity usually not well defined. The

inconsistency was thought to be due to density flows - either because of the tracer density or possibly temperature changes in the lake water.

The majority of the experiments and usable results to date have been made using either Rhodamine B as the tracer and a fluorometer (G.K. Turner Associates-model 111) to detect outflow concentrations or dilute solutions of ammonium hydroxide and conductivity probes. In the first case, a continuous sample is subject to ultraviolet light causing the sample to fluoresce in proportion to the concentration of Rhodamine B present. The fluorescent light is then detected on an optical bridge which can be read at particular times or continuously recorded on a DC strip chart recorder. In order to operate the instrument in the most sensitive mode, light filters particular to the tracer being used must be obtained. Using the recommended filters, it is no problem to detect concentrations of .01 ppm in the outflow. Rhodamine B, injection concentration 100 ppm, mixes well with the Detroit River inflow and does not exhibit density flow characteristics.

The conductivity probes were made, using approximately 1 mm square copper electrodes spaced about 1 mm apart. The change in local conductance in the neighborhood of the probe was sensed by the probes, amplified and recorded on an oscillograph. Early versions of the probe with larger electrodes were found to be sensitive to changes in depth of submergence. This aspect was minimized but care still had to be taken in their use. The dilute solutions of ammonium hydroxide (< 2%) were only slightly buoyant and this difference in density was in most cases negligibly small. Reproducibility of results were very good utilizing this technique.

Initial experiments with Rhodamine B were designed to measure the detention time only and so quantity control was lax, relative concentration being satisfactory. The fluorometer was positioned so that part of the outflow passed through the fluorometer and back to the recirculating sump. With this experimental arrangement, it was not possible to determine dilution factors or perform a mass balance on the tracer. Experimental accuracy in determining the time to peak concentration was limited by speed of the recorder and injection technique. Injection took place within 20 seconds and chart speed was checked and found accurate to $\pm$ 1 min/hour so that overall accuracy of time measurement is $\pm$ 2%.

Later experiments were conducted by short circuiting and passing the entire flow through the fluorometer in addition to measuring the time span over which the Rhodamine B was injected into the flow. Injection was accomplished using a needle valve metering cup. The valve can be set so that the tracer is added over a measurable time period into the inflow. The point of injection was well upstream in the Detroit River and the tracer was well mixed by the time the flow reached the lake. A hypodermic needle was loaded with 2.00 $\pm$ .05 cc of 0.1000 $\pm$ .0001 g/L of Rhodamine B solution prepared using an analytical balance. The solution is discharged into a metering cup and the time from first show of tracer to last show of tracer observed with a hand operated stop-watch. The greatest possibility for error exists at this point because (1) it is difficult to tell when the last show occurs since it is a drop by drop process (estimate a 5% time error) and (2) some of the Rhodamine B coats

and remains inside the cup (estimate a 10% quantity error). These errors manifest themselves in the calculation of incoming tracer concentration causing, in the worst possible combination, a concentration error of 14%. The entire outflow is passed through a fluorometer and concentrations recorded on a DC strip-chart recorder. Recorder output is accurate to 2% in scale and 2% in time.

IV.4 Seiche Studies

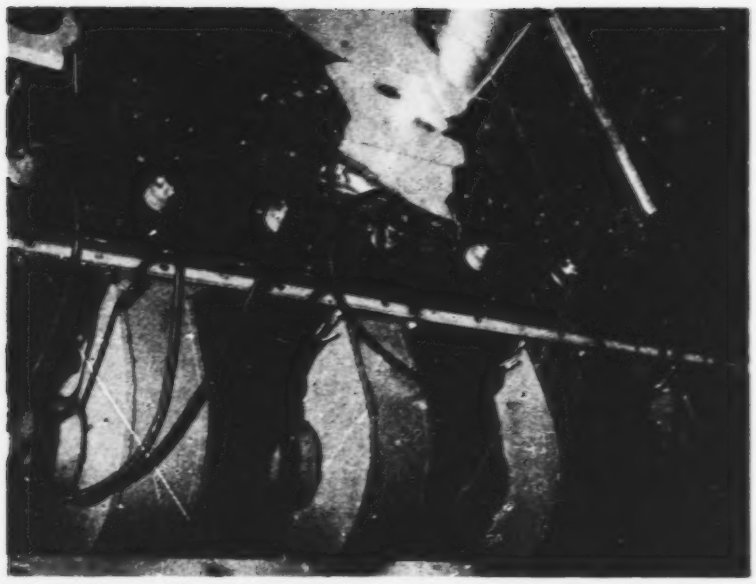
Seiches are artificially generated in the model by raising one end of the model lake a short distance, holding the model in a raised position until the entire water mass has come to rest, and then smoothly and rapidly lowering the model. Seiches were always generated by raising at Buffalo and tilting about a North-South axis. Experiments with other axes of rotation produced no detectable differences (within instrument range) in seiche characteristics.

Although motion pictures of surface particle movements have been made, the primary method of observing a seiche has been with capacitance type wave gages. These gages consist of 2" dia. copper plates connected to circuitry capable of detecting pico farad changes in capacitance, amplifying this change and producing a DC analogue output. Two detection devices have been used; one which was modeled after a circuit by Phelps and built at SUNYAB and the other a commercially manufactured capacitance-inductance meter (Boonton Electronics Corp. model 71A). Both devices are capable of detecting water surface level changes of 0.001 inch but for this level of detection a price must be paid. The price is that calibrations of water surface movement against recorder response must be run frequently to guard against mean drift of scale factor changes.

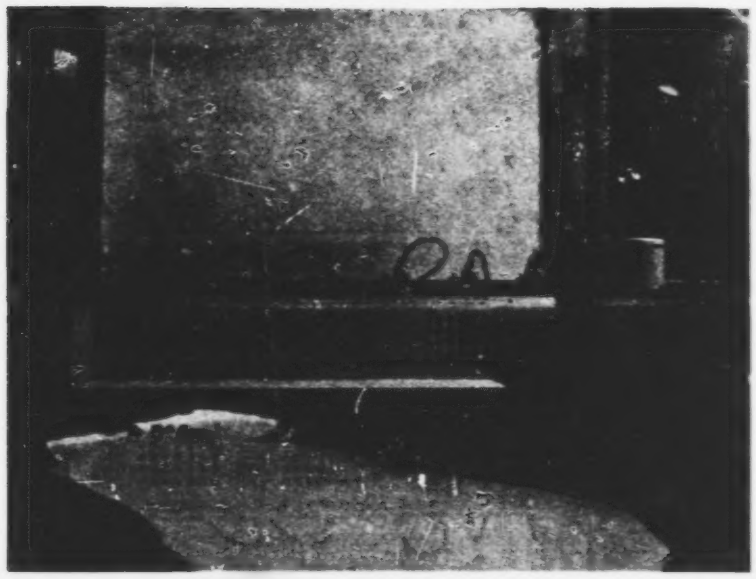
By attaching the capacitor plate to a micrometer barrel (least count .001") it is possible to move the gauge up and down known increments of distance and so effect a rapid calibration. Standard procedure consisted of performing the calibration, running 5 seiche cycles and calibrating again, all within a time span of 30 minutes. The capacitance gauge has the advantage of not touching the water and the disadvantages of non-linearity; the closer the capacitor plate is to the water surface the greater the change in capacitance for a given change in water surface elevation. Care had to be given to two sources of error which were oscillations induced by the axis of rotation not being perfectly vertical and difficulty in determining the mean water height about which the seiche oscillation occurs. A dual channel DC strip chart recorder (Mosley, model HP 7100B) was used to simultaneously record the output of both gauges. The speed of this recorder was found accurate to $\pm$ 1%.

IV.5 Circulation Studies With Wind

During all recorded experiments, the model lake was subjected to a prevailing westerly wind. A battery of four centrifugal blowers, each with a rated capacity of 350 cubic feet per minute, provided the westerly wind. As shown in Figs. 6 and 7, the blowers fed into a diffuser equipped with a wooden grid work which was intended to reduce the nonuniformity of the wind. Examination of the wind speed profiles shown in Figs. 8 and 9 reveals that a good portion of the nonuniformity of the wind was smoothed out by the diffuser. The wind speed was controlled by a variable transformer (variac) which regulated the speed (or discharge) of the blowers. Actual model wind speeds could be varied from zero to an average of 13 feet per second.



Photograph of Blowers and Plexiglass Top



Looking Upwind Over Lake Surface

Fig. 6 Photographs of Experimental Setup for Wind Driven Studies

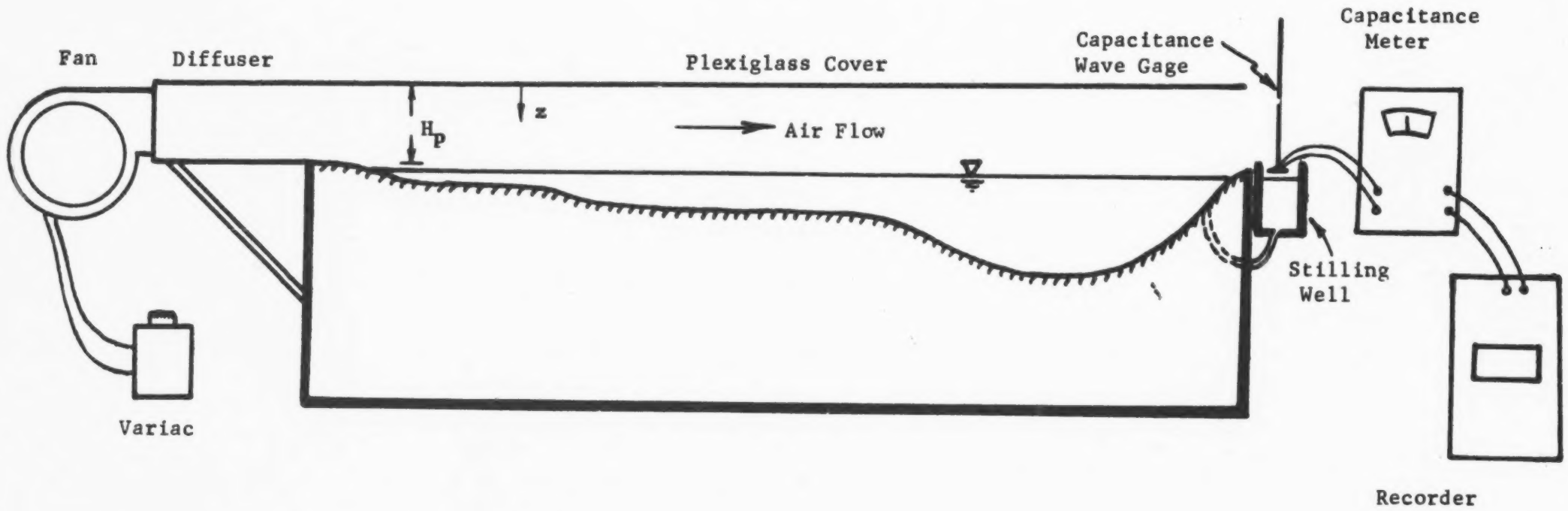


Fig. 7 Experimental Setup Used in Wind Driven Circulation Studies

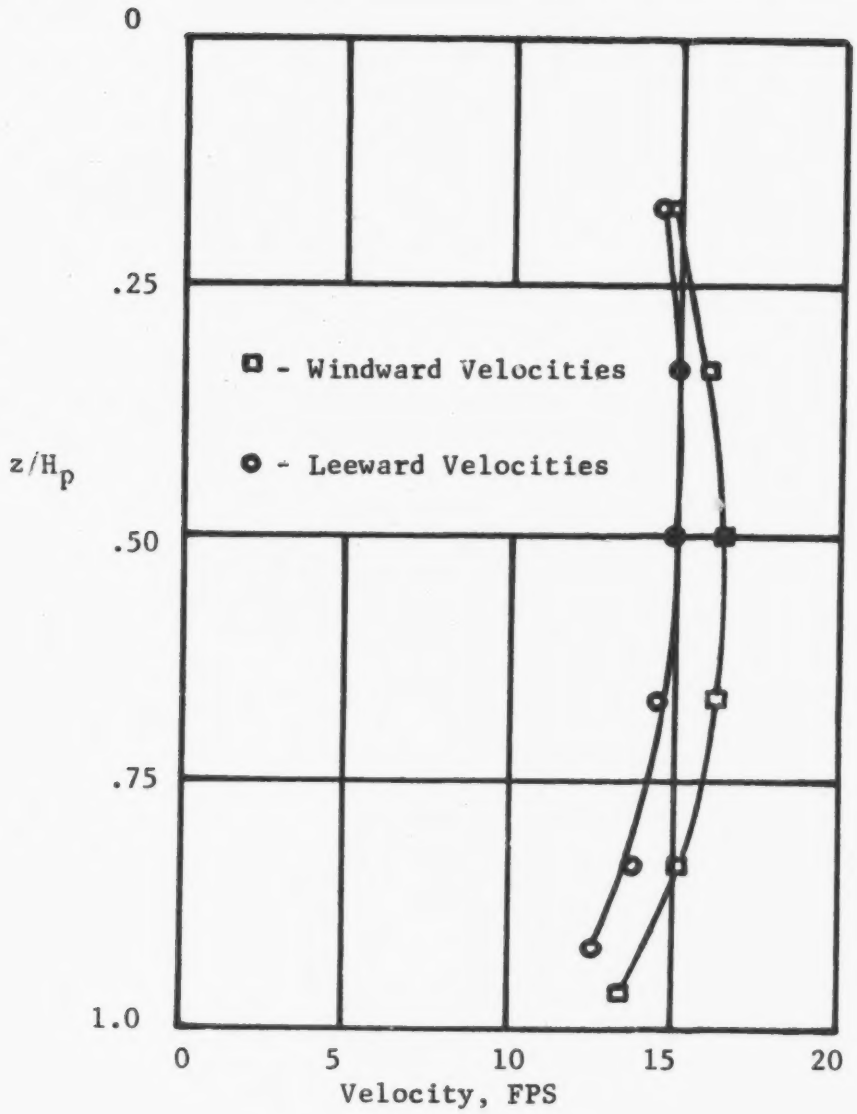


Figure 8

Vertical Distribution of Model Wind Velocities

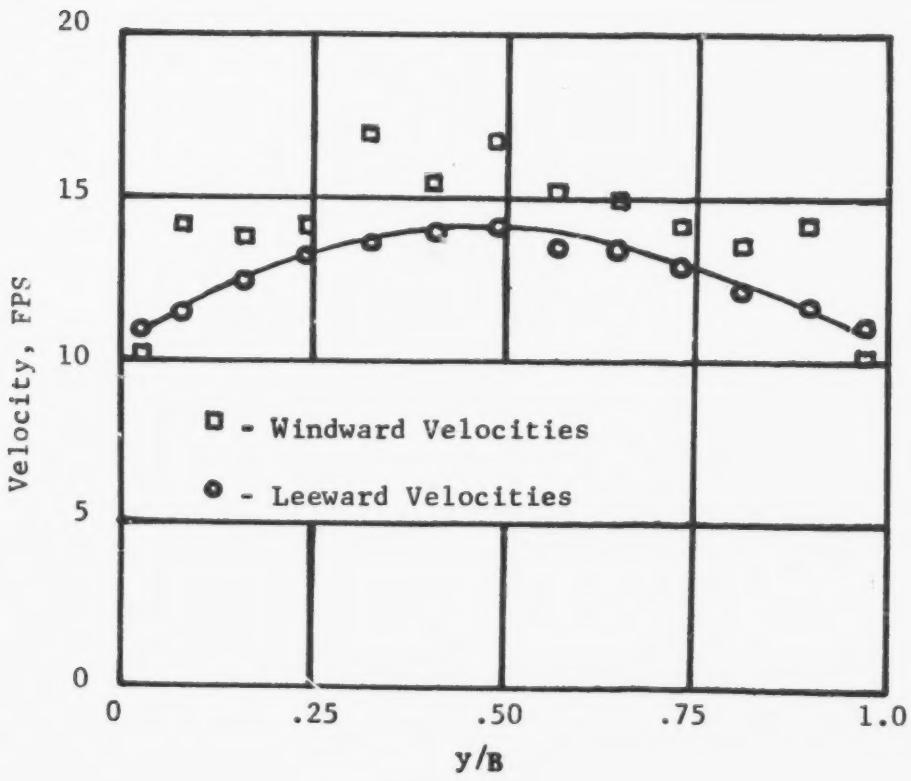


Figure 9

Horizontal Distribution of Model Wind Velocities

A plexiglass cover 6 inches high and 37 inches wide was used to contain the wind over the model lake and prevent it from diffusing. A series of 3/4 inch holes were drilled in the plexiglass cover to permit access to the lake when the cover was in place.

Measurements were made of model water level changes using the capacitance wave gage in conjunction with a strip chart recorder. This experimental arrangement measured water level changes to $\pm$ 0.001 inches. The presence of capillary waves on the model water surface made measurement of water level changes somewhat difficult. This difficulty was resolved satisfactorily through the use of a stilling well connected to the far eastern end of the model.

Circulation patterns were established in two ways. Motion picture records of the continuous injection of tracer at the mouth of the Detroit River were made and studied. Gross circulation patterns could be established in this way. In addition, small quantities of tracer were injected at various locations along the perimeter of the model lake to establish the finer details of the general circulation pattern.

Two types of tracer were used in this study; neutrally buoyant dyes and finely powdered aluminum particles. The dyes were used in the establishment of the gross circulation pattern and the finer details of the subsurface circulation pattern. The finer details of the surface circulation pattern were established with the aluminum particles.

Several experimental difficulties and limitations to the present set-up became apparent during the course of the study. The model lake was constructed with relatively high boundaries along its perimeter. These boundaries appeared to influence the distribution of wind stress over the model lake. Because of their probable effect, the boundaries

surrounding the lake were trimmed down so that the wind stress could be applied more evenly to the water surface. The limitations of the diffuser became apparent during the wind velocity measurements. A completely uniform wind profile could not be achieved with the present diffuser. A more restrictive diffuser would have further smoothed out the wind profile, but would have decreased the wind velocity significantly. Blowers with higher capacities would permit the use of a better diffuser and have the added advantage of being able to simulate stronger prototype wind velocities.

The dominant forces in lake models are due to gravitational accelerations, Coriolis acceleration, wind stresses, and boundary resistance. The requirements for similitude of the forces due to these natural phenomena have been discussed thoroughly by Harleman et al (1962) (1964). The general requirements of equality of Froude number and Rossby number in model and prototype insure similarity of gravitational and Coriolis effects on horizontal flow. Adjustment of boundary resistance can be accomplished through the process of model verification. A more difficult modeling problem is the simulation of prototype wind stress at the water surface in the model lake system. This aspect of the modeling problem has been discussed by von Arx (1959). The actual stress applied to a water surface is a combination of both tangential shear stress and the pressure or form drag that develops when the surface has wave forms. The usual way to portray this resultant wind stress is by an equation of the form

$$\tau_s = \tfrac{1}{2}\rho C_D V^n \qquad (4\text{-}2)$$

where τ_s is the wind stress, ρ the density of the air, C_D the

coefficient of drag, V equals the average wind speed at some appropriate distance above the water surface, and n is an exponent normally considered to be 2. Although considerable study has been given to the problem of momentum transfer across an air-water interface, the ability to accurately predict C_D is still subject to question. For this reason empirical procedures have been used in determining the scaling relationship between model and prototype wind speeds. One technique recommended by von Arx (1957) and used by Harleman et al (1964) is the procedure of adjusting wind velocities in the model until surface currents are in scale with field observations in the prototype.

A different approach was employed in this study. This approach is based on the response of the water surface profile (wind tide or wind set-up) to the wind stress. When a wind stress is applied to the water surface of a basin, water on and near the surface is transported from the windward to the leeward end of the basin. This movement causes a rise in the water surface elevation at the leeward end and a drop in the surface elevation at the windward end of the basin. In other words, the water surface is given a positive slope when viewed from the windward end. The amount of water level change or wind set-up is a function of the wind speed and direction, the length of the basin (or fetch), the depth of water in the basin, and the temperature difference between the water and the air. Wind set-up is a direct function of wind speed. Knowledge of this function enables predictions of wind speeds to be made from known wind set-ups. This technique, often used in real lakes, was applied to the present model study.

The change in water level at the Buffalo end of the model was measured for various model wind speeds. These wind tides or set-ups

were scaled up to prototype dimensions according to the vertical scale ratio of 1:500. Using prototype data [Hydro-Electric Power Commission of Ontario] that related water level changes from calm at Buffalo for varying wind speeds and directions, it was possible to assign a prototype wind speed for the model experiments.

Keulegan (1961) has conducted analytical and experimental studies on the wind set-up for smooth and rough water surfaces. In his report, Keulegan gives the following expression for calculating the difference in water level between the extreme ends of a lake as

$$\frac{S}{L} = 3.30 \times 10^{-6}\left[1 + 63 \frac{H^{\frac{1}{2}}}{L}\right] \frac{V^2}{gH} \qquad (4-3)$$

where S is the set-up or difference in water level between extreme ends, L the length of the basin, H the average depth, and V the wind speed. This expression will be used for comparison later in this report.

Keulegan along with Baines and Knapp (1965) discuss the requirements for dynamic similarity when modeling wind flow over a water surface. The Reynolds number based on shear velocity at the free surface is suggested as the important modeling parameter. Just as bottom frictional effects in a distorted model are determined by the process of model verification, it appears that the water surface frictional effects must be also empirically scaled.

V. Summary

This first report contains information relating to the philosophy and justification for the use of a hydraulic model in the study of water movement in Lake Erie. The experimental set-up has been described and

the experimental procedures outlined. A second report (Part II) will present the results obtained from this model study and discuss these results and their relationship to mathematical models and prototype observations. A complete set of drawings exists which provides the information necessary for the fabrication and erection of the rotating laboratory. Copies of these drawings can be obtained upon request. Their size and bulk precluded attaching them to this report.

The operation of this laboratory has proven entirely satisfactory. When rotating, the laboratory is free of vibrations. Occasionally the position of the upper bearing needs adjustment so that the laboratory rotates in a true horizontal plane. No difficulties have been encountered with the electrical service to the laboratory. Although provision for heating lamps to simulate solar heating has been provided, the installation of these lamps has not yet been accomplished.

The experience of the researchers working in the laboratory indicates that motion sickness can be a problem for some initially. This problem seems to disappear as one becomes conditioned to working in the rotating laboratory.

VI References

Baines, W.D and Knapp, D. J. (1965), "Wind Driven Water Currents," Proc. ASCE, Vol. 91, No. HY2, pp. 205-221

Buechi, P. and Rumer, R., (1969), "Wind Induced Circulation Pattern in a Rotating Model of Lake Erie," Proc. 12th Conf. Great Lakes Res. Internat. Assoc. Great Lakes Res., pp. 406-414

Carr, J. (1962), "Dissolved Oxygen in Lake Erie, Past and Present," Pub. No. 9, Great Lakes Res. Div., Univ. of Mich., pp. 1-14

Carr, J., Beeton, A. and Allen, H. (1963), "Factors Associated with Low Dissolved Oxygen Concentration in Lake Erie," Pub. No. 10, Great Lakes Res. Div., Univ. of Mich., p. 133

Chabert-D'Hieres, G. and Hyacinthe, J.L. (1967), "Similitude des Phenomenes de Diffusion Geophysique," La Houille Blanche, No. 6 pp. 659-666

Chappell, T. (1963), "An Investigation of the Establishment of a Thermocline in a Model Lake," M.S. Thesis, M.I.T., Cambridge, Mass. 48 p.

Csanady, G. T. (1964), Turbulence and Diffusion in the Great Lakes," Pub. No. 11, Great Lakes Res. Div., Univ. of Mich., pp. 326-339

Harleman, D. R. F., Bunker, R. and Hall, J. (1964), "Circulation and Thermocline Development in a Rotating Lake Model," Pub. No. 11, Great Lakes Res. Div., Univ. of Mich., pp. 340-356

Harleman, D. R. F., Holley, E. Hoopes, J. and Rumer, R. (1962), "The Feasibility of a Dynamic Model of Lake Michigan, " Pub. No. 9, Great Lakes Res. Div., Univ. of Mich., pp. 51-67

Hoopes, J. A. (1969), Personal Communication

Hydroelectric Power Commission of Ontario, Unpublished Data on Lake Erie Set-Up

Keulegan, G.H. (1961), "Wind Tides in Small Closed Basins," Journ. Nat. Bur. Stds, Vol. 46, No. 5, pp. 358-381

"Lake Erie Environmental Summary," (1968), FWPCA Report, U.S. Dept. of Interior, 170 p.

Liggett, J. A. (1969), "Unsteady Circulation in Shallow Homogeneous Lakes," Proc. ASCE, Vol. 95, No. HY4, pp. 1273-1288

Orlob, G. T. (1959), "Eddy Diffusion in Homogeneous Turbulence," Proc. ASCE, Vol. 85, No. HY9, pp. 75-101

Platzman, G. and Rao, D. B. (1963), "The Free Oscillations of Lake Erie," Tech. Report No. 8, Dept. of the Geophysical Sciences, Univ. of Chicago

Rumer, R. and Robson, L. (1968), Circulation Studies in a Rotating Model of Lake Erie," Proc. 11th Conf. Great Lakes Res., Internat. Assoc. Great Lakes Res., pp. 487-495

Taylor, G. I. (1953), "Dispersion of Soluble Matter in Solvent Flowing Slowly through a Tube," Proc. Roy. Soc. (London) A219, pp. 186-203

von Arx, W. S. (1954), "A Laboratory Model of the Wind Driven Ocean Circulation," Weather, Vol. 9, pp. 177-184

von Arx, W. S. (1957), "An Experimental Approach to Problems in Physical Oceanography," Physics and Chemistry of the Earth, Pergamon Press, Vol. 2, pp. 1-29

von Arx, W. S. (1962), An Introduction to Physical Oceanography, Addison-Wesley Publishing Co., Reading, Mass.

END